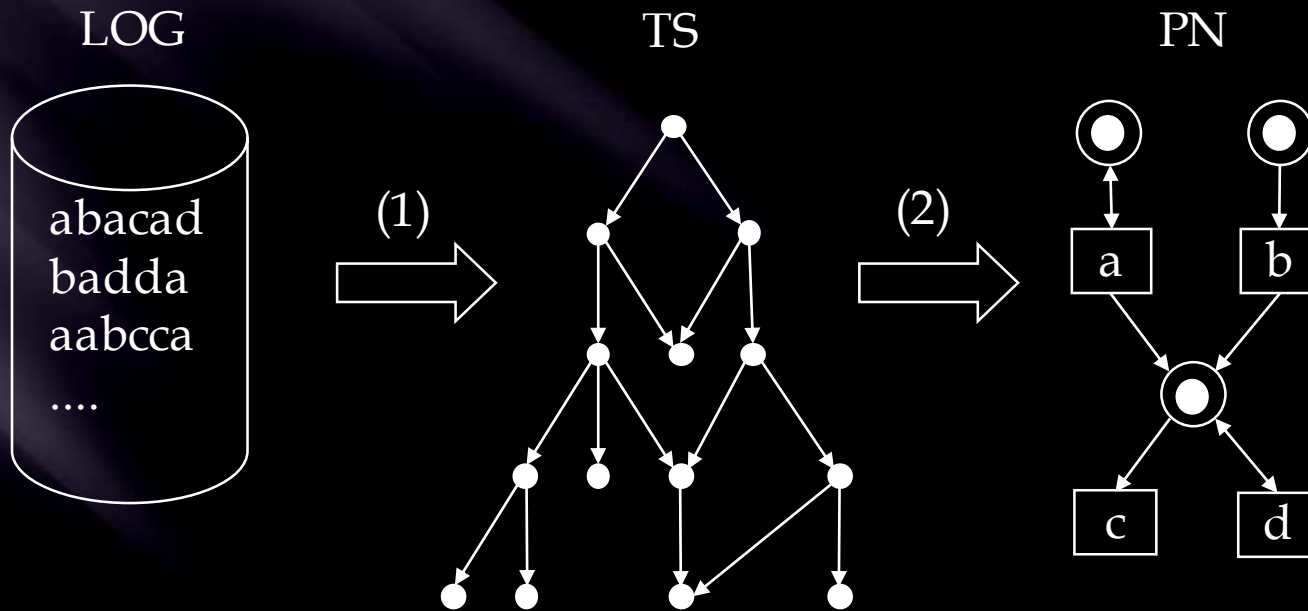


Outline

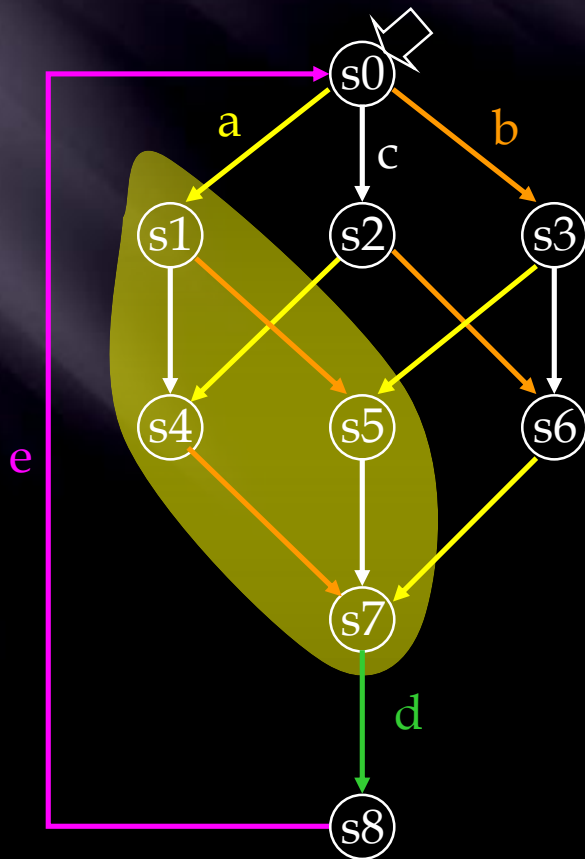
- ▣ **Region-based approach for Process Mining**
- ▣ Decompositional strategy: sequential views
- ▣ Divide-and-Conquer strategy

From logs to TS



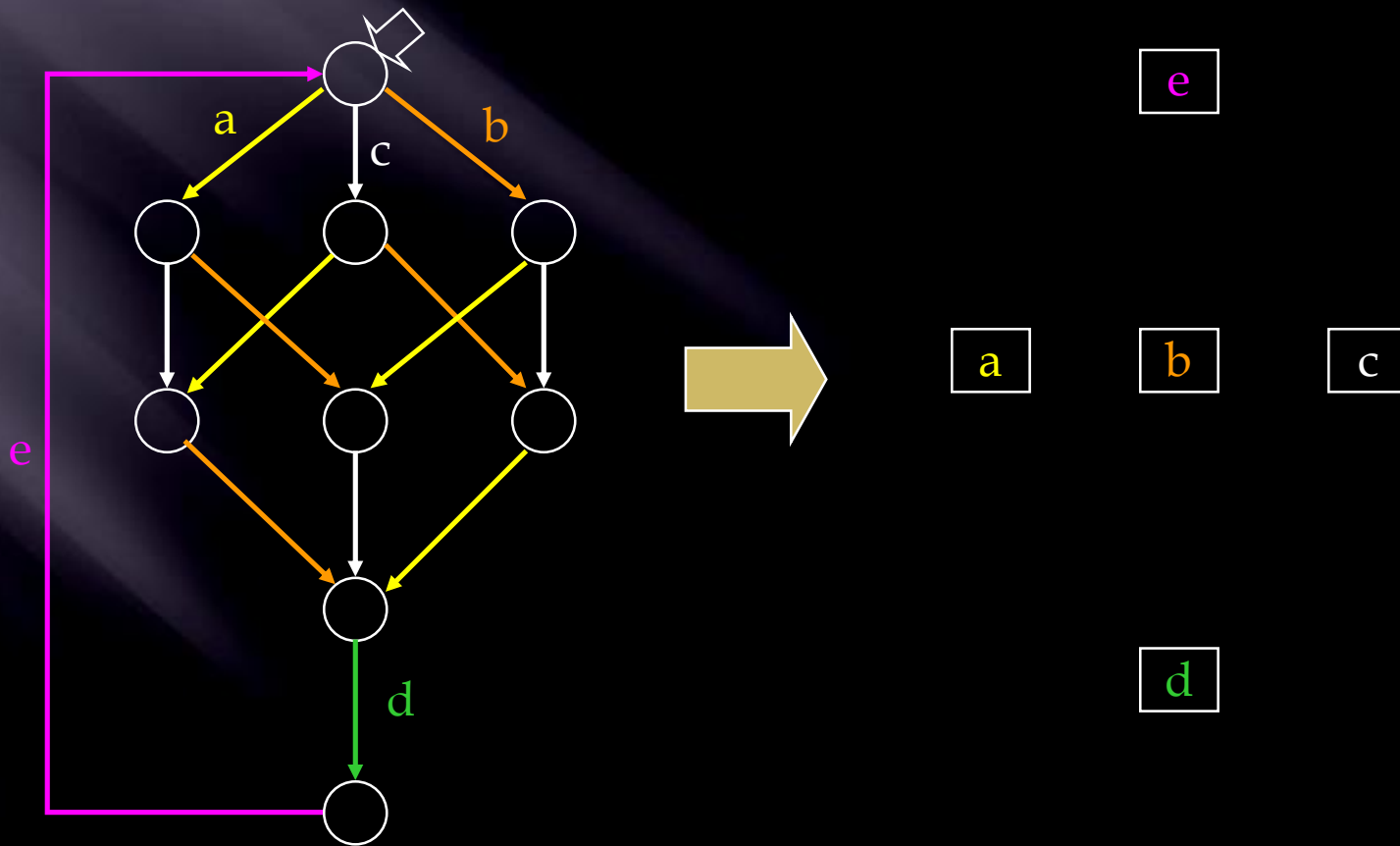
Region [Ehrenfeucht & Rozenberg]

A set of states is a region if every event has an homogeneous relation with the region



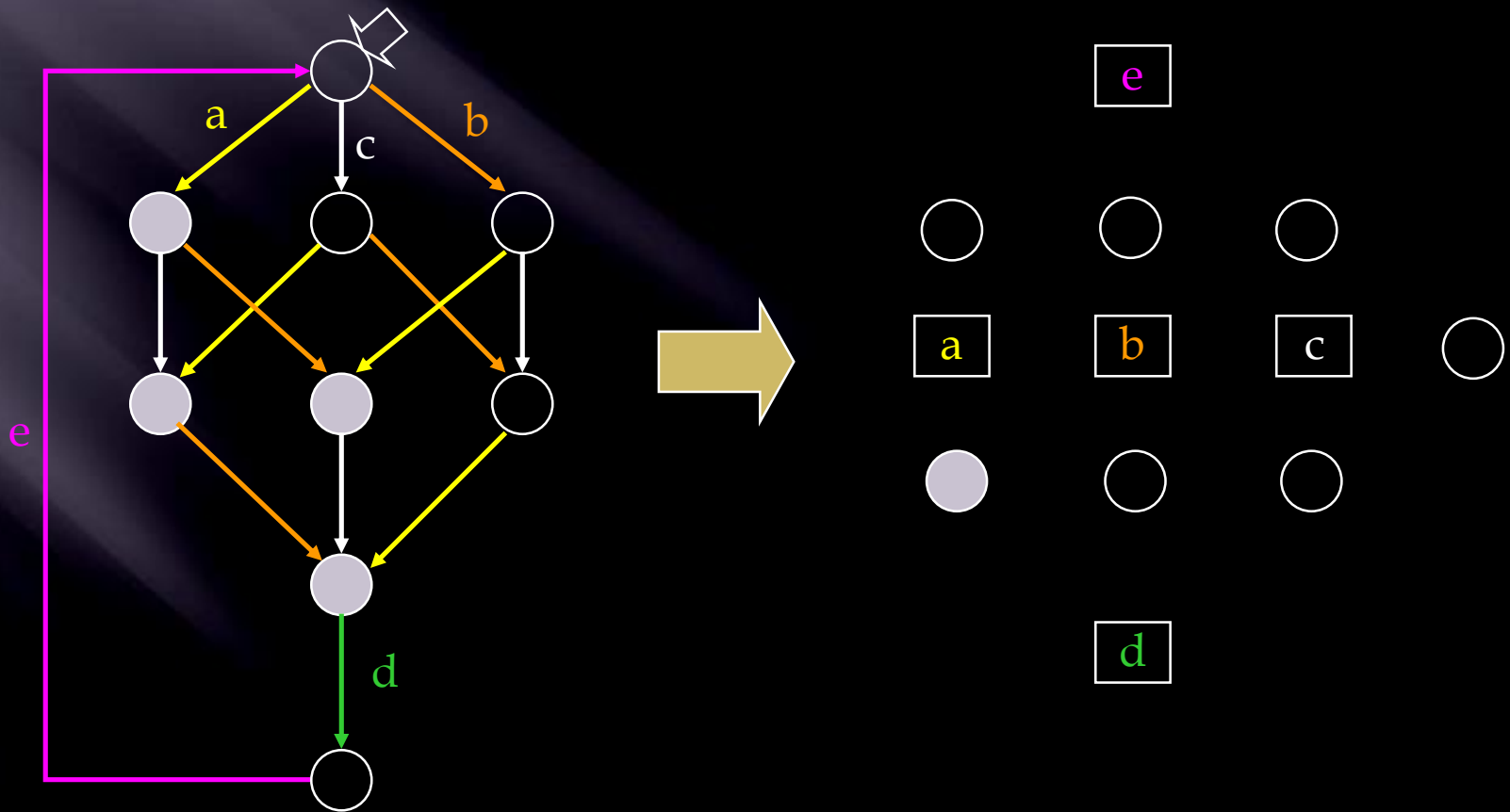
- $S = \{s_1, s_4, s_5, s_7\}$
 - a enters S
 - d exits S
 - c, b and e do not cross
- Regions correspond to PN places

PN synthesis [Ehrenfeucht & Rozenberg]



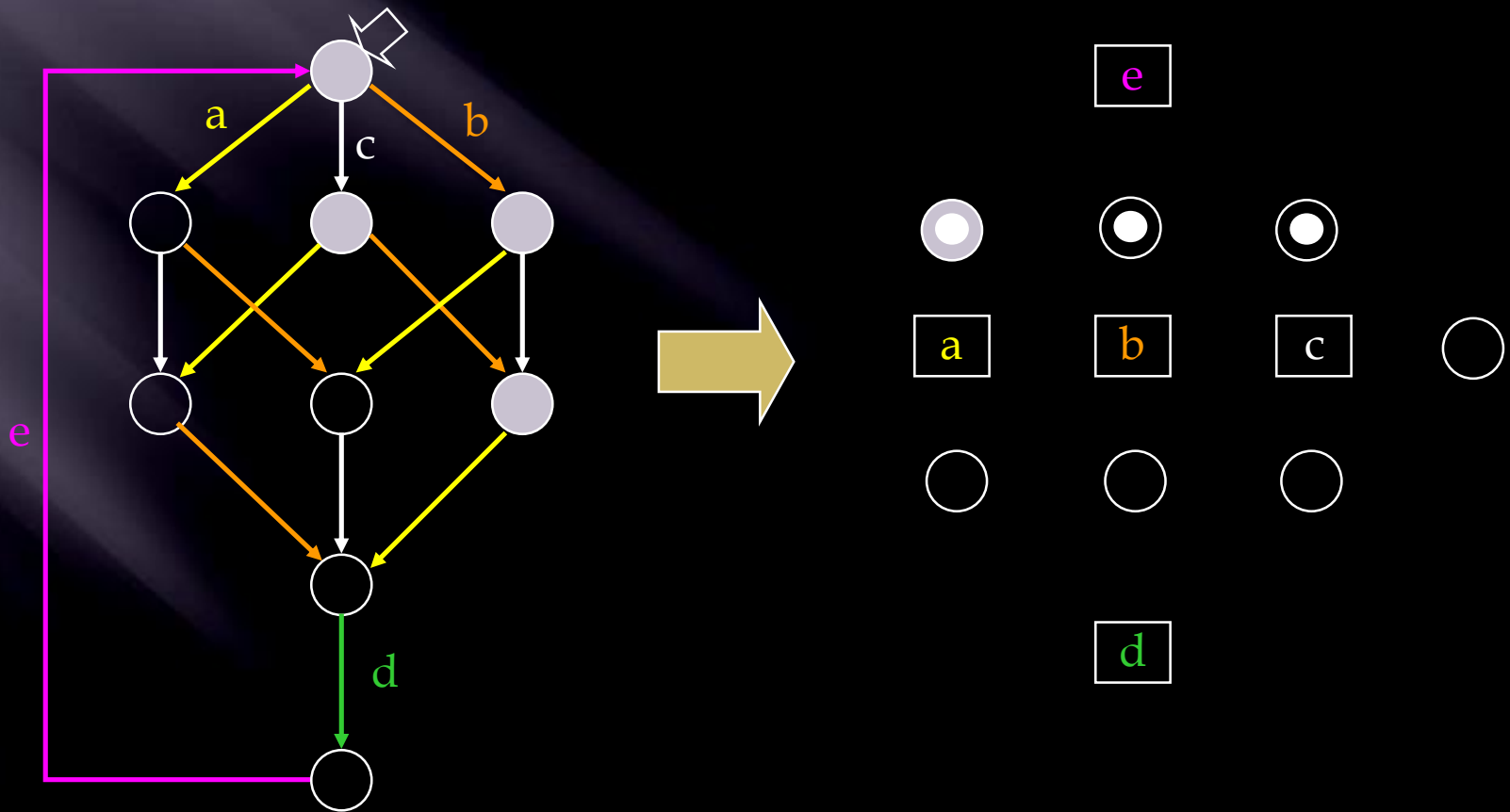
For every event generate a transition labeled with this event

PN synthesis algorithm



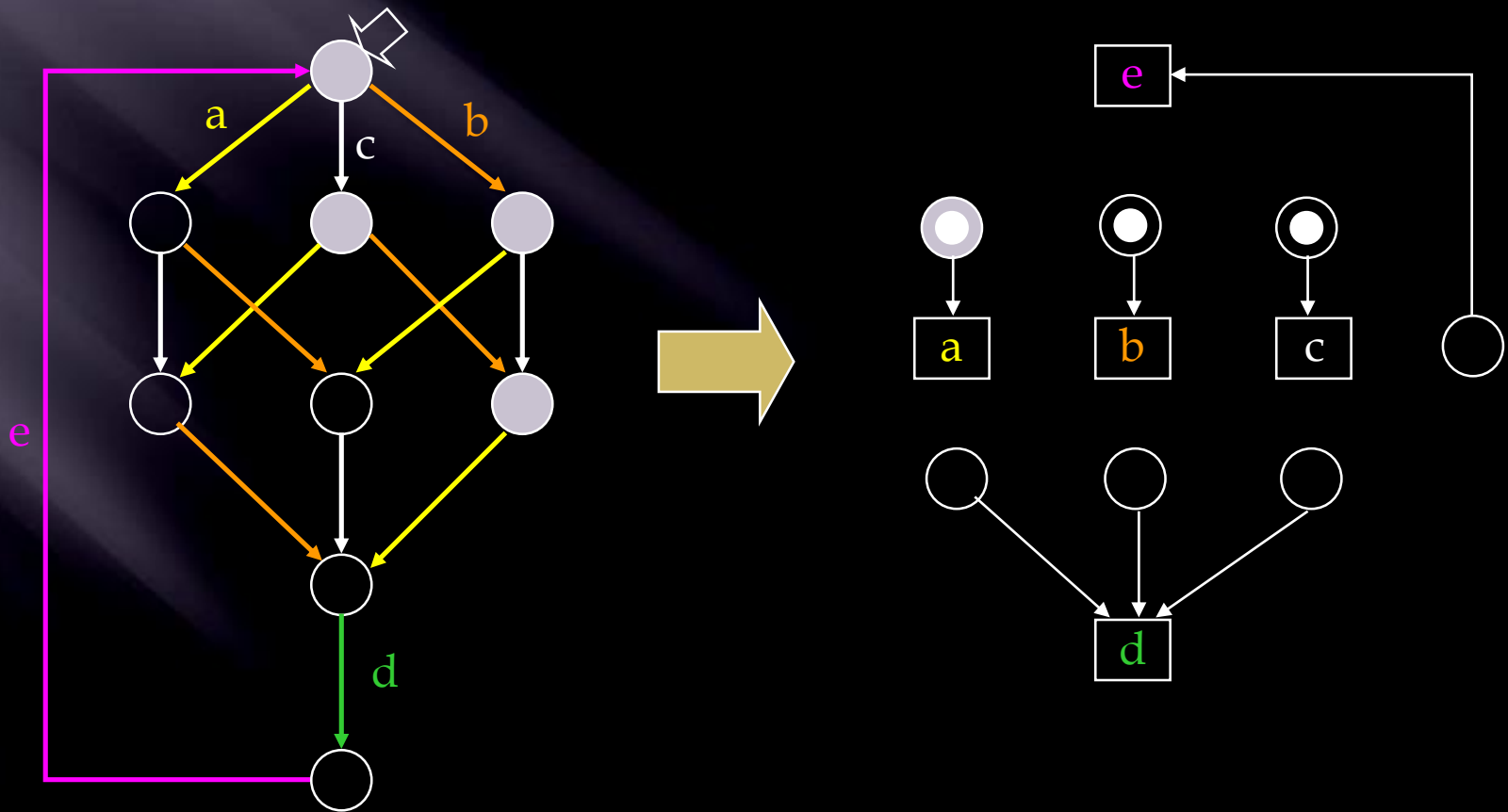
For every (minimal) region generate a place

PN synthesis algorithm



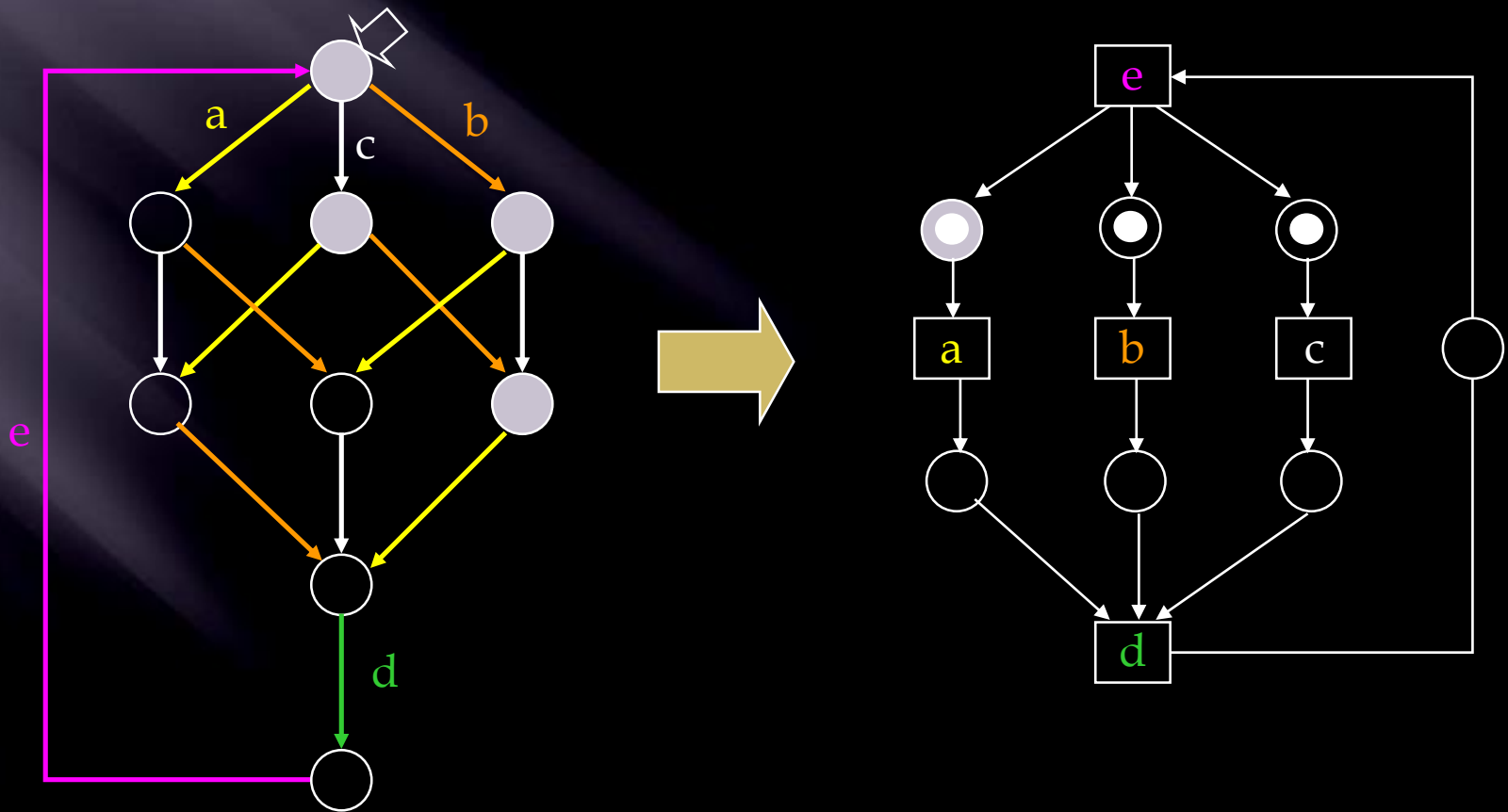
If region includes the initial state then mark the corresponding place

PN synthesis algorithm



If event exits the region add an arc between the corresponding place and the corresponding transition

PN synthesis algorithm



If event enters the region add and arc between the corresponding transition and the corresponding place

Limitations

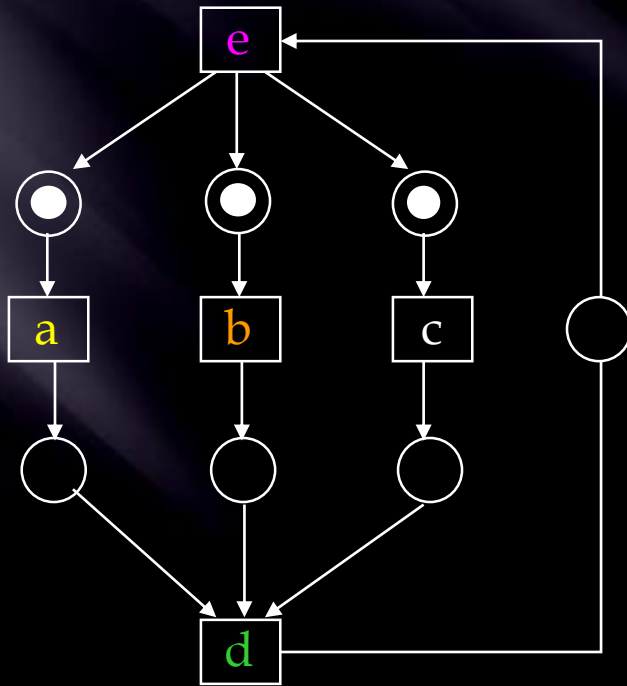
- ▣ Large TSs cannot be handled by this approach
- ▣ The use of efficient data structures and algorithms may help, but the problem still remains
 - Implicit representation (Binary Decision Diagrams)
 - Dynamic Programming
 - Exploration heuristics
- ▣ Language-based regions [Bergenthum *et al.*, Van der Werf *et al.*] suffer from the same problem

In the rest of the talk ...

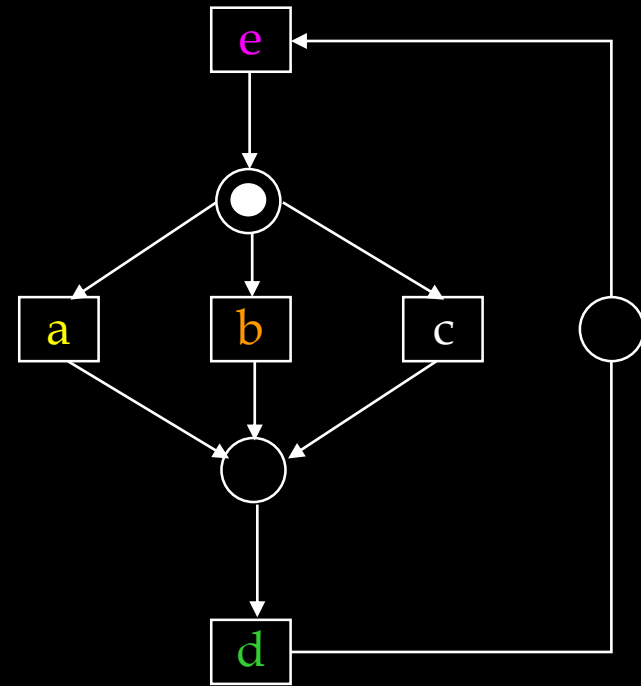
- ▣ Two strategies to overcome these limitations:
 - 1) *Decompositional approach*: search for a set of **sequential** views of the log that jointly explain it
 - 2) *Divide-and-Conquer approach*: iteratively **project** the log into smaller logs that can be handled afterwards

Decompositional Strategy

- Sequential views of the log \Leftrightarrow State Machines (SM)



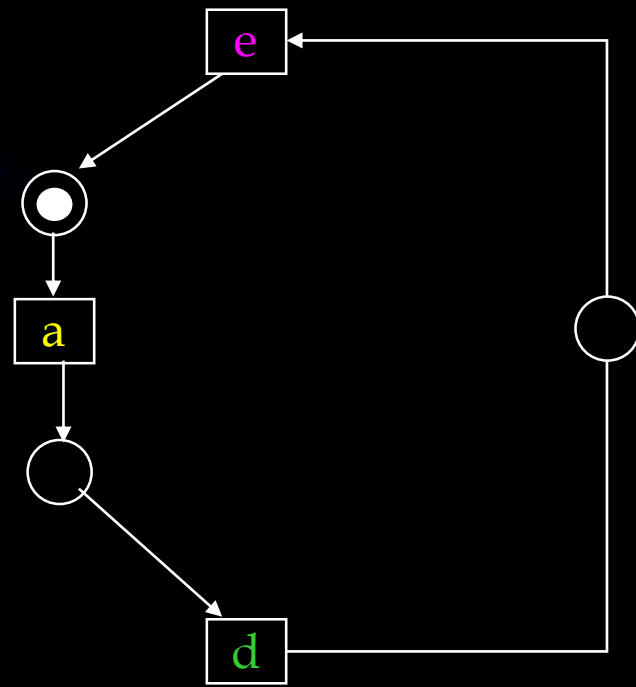
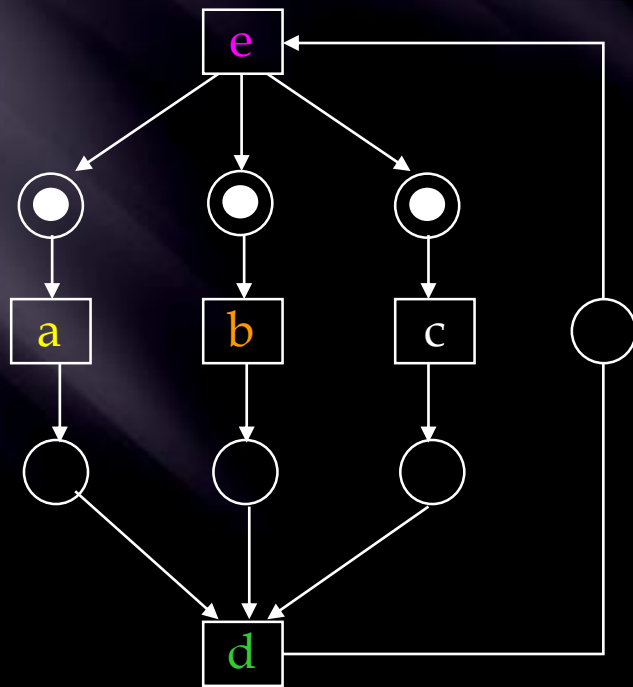
Not an SM: a,b,c concurrent



SM

Decompositional Strategy

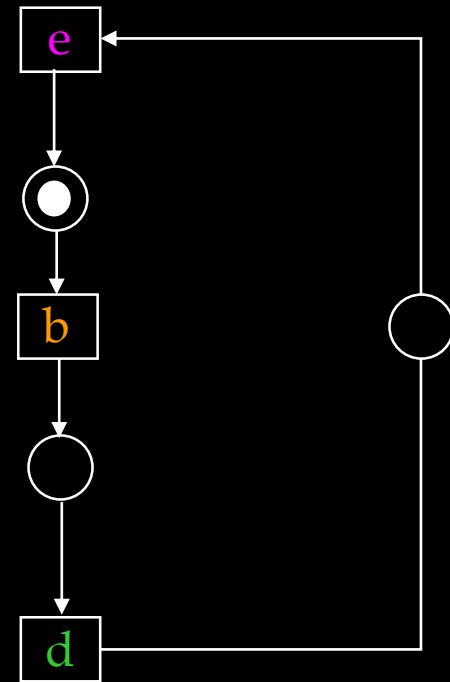
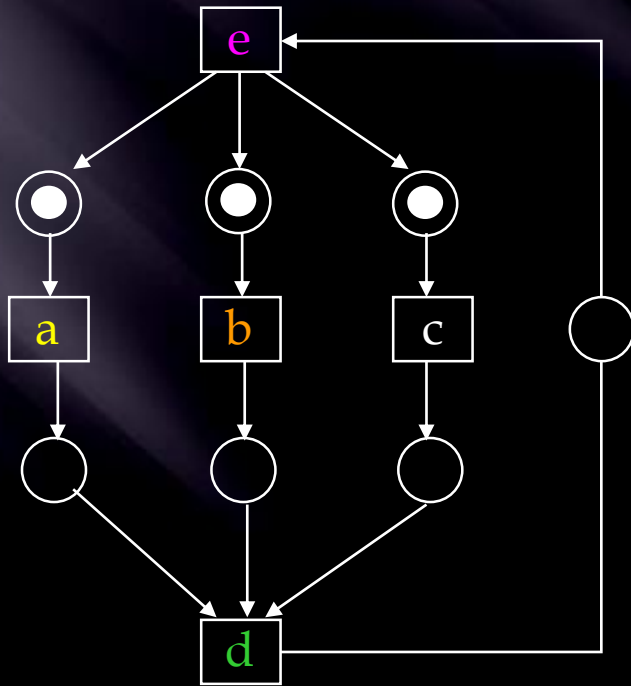
- State-machine component of a PN:



SM1

Decompositional Strategy

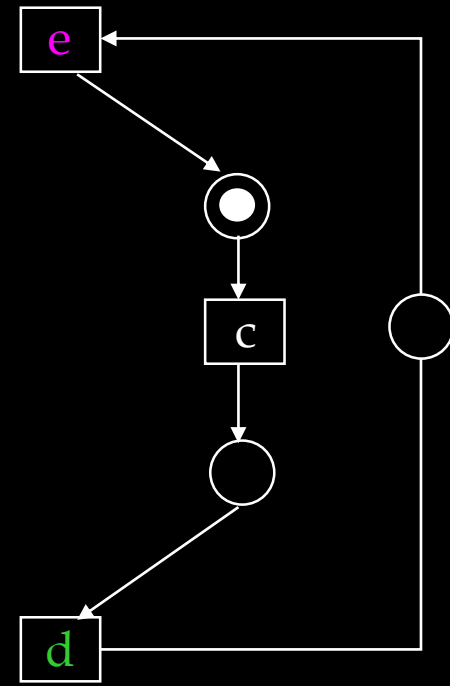
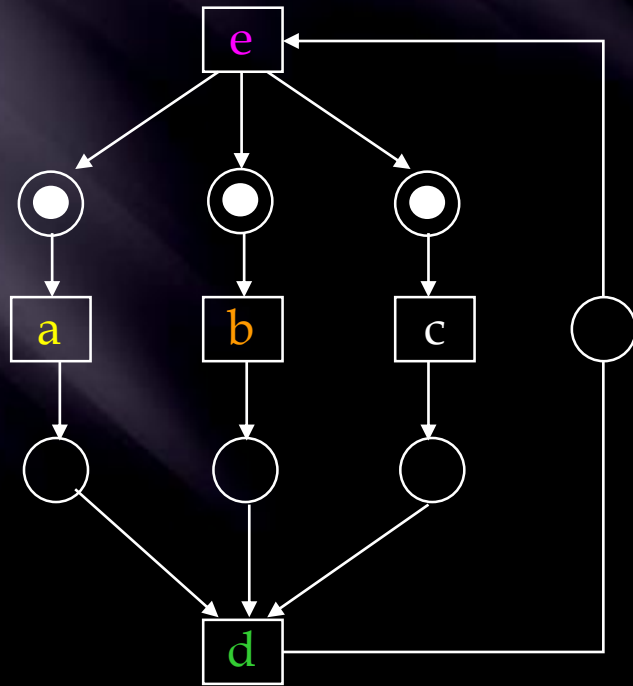
- State-machine component of a PN:



SM2

Decompositional Strategy

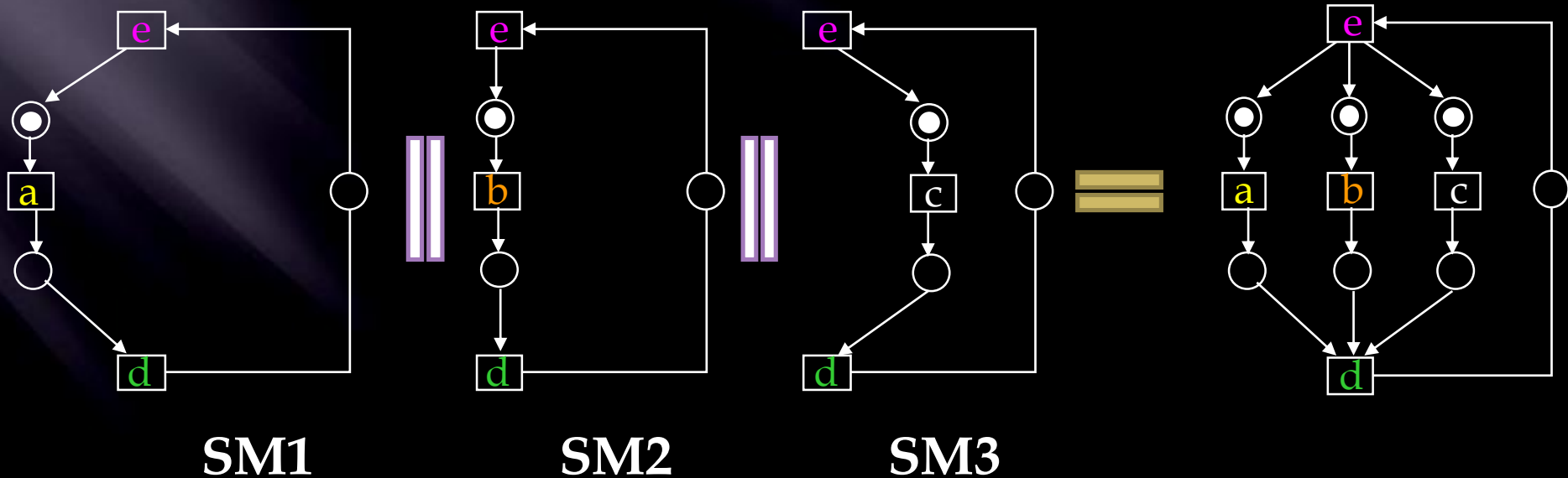
- State-machine component of a PN:



SM3

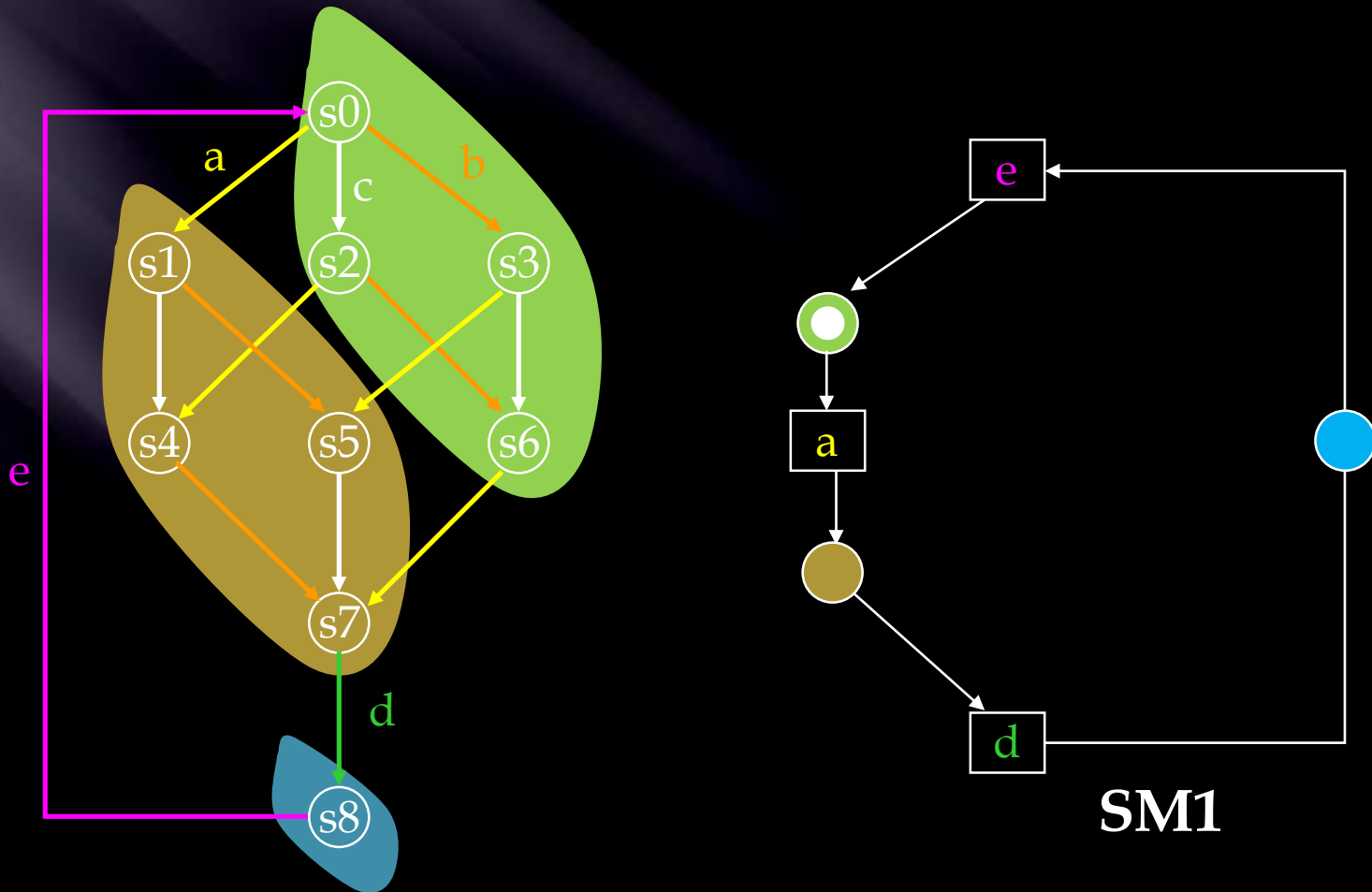
Decompositional Strategy

- Parallel composition:



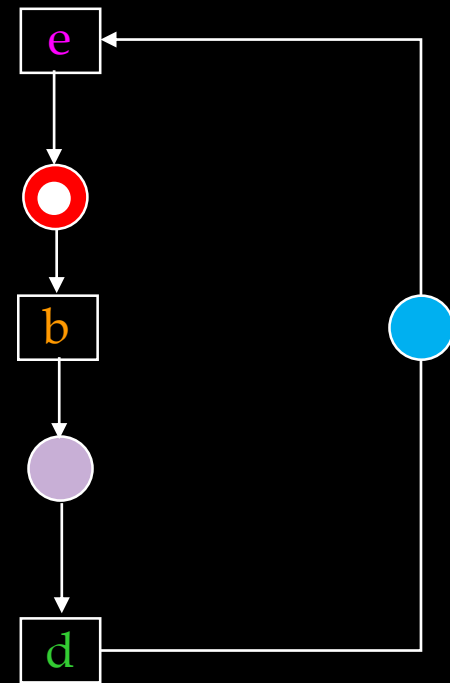
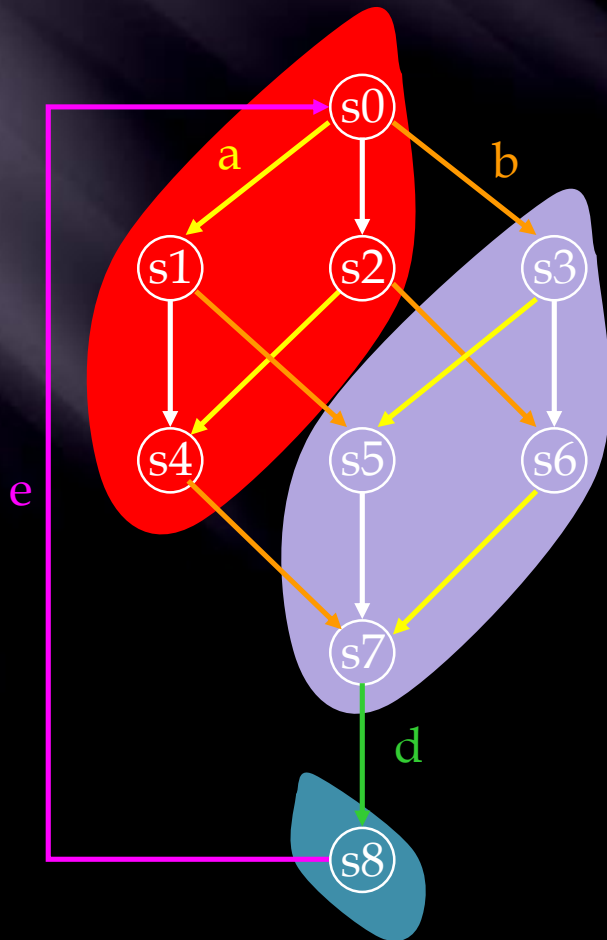
Decompositional Strategy

Theorem: a **partition** of a TS by regions is a SM



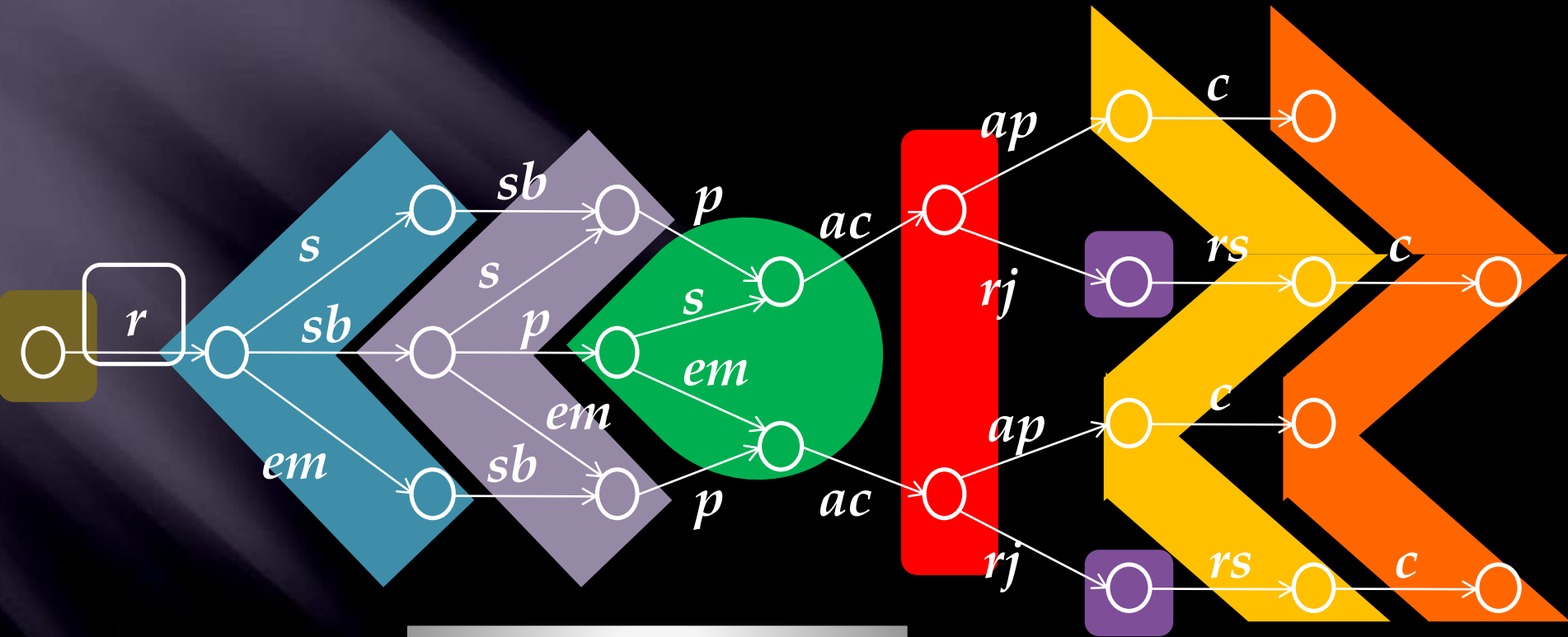
Decompositional Strategy

Theorem: a **partition** of a TS by regions is a SM

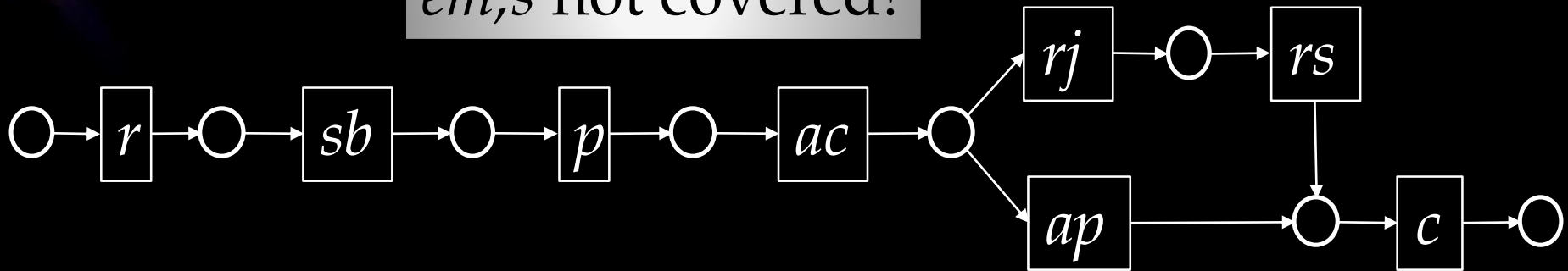


SM2

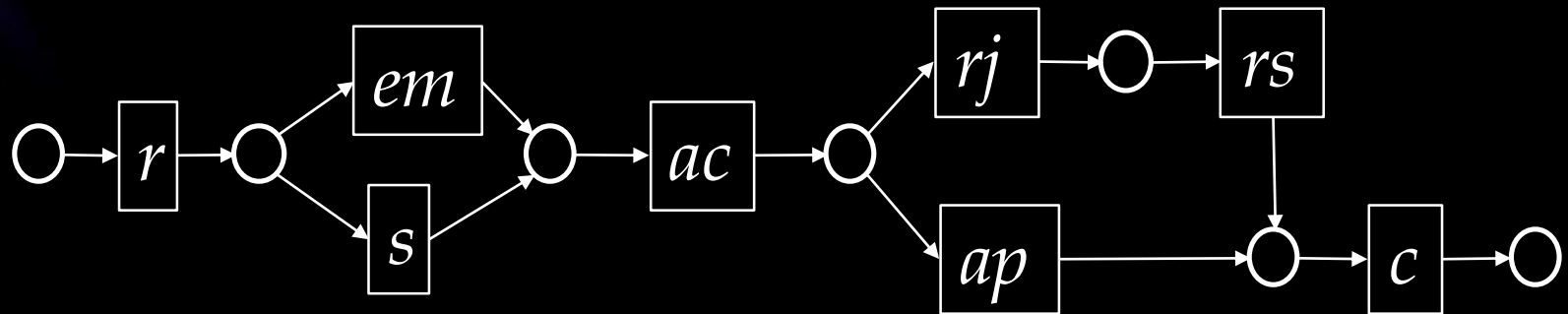
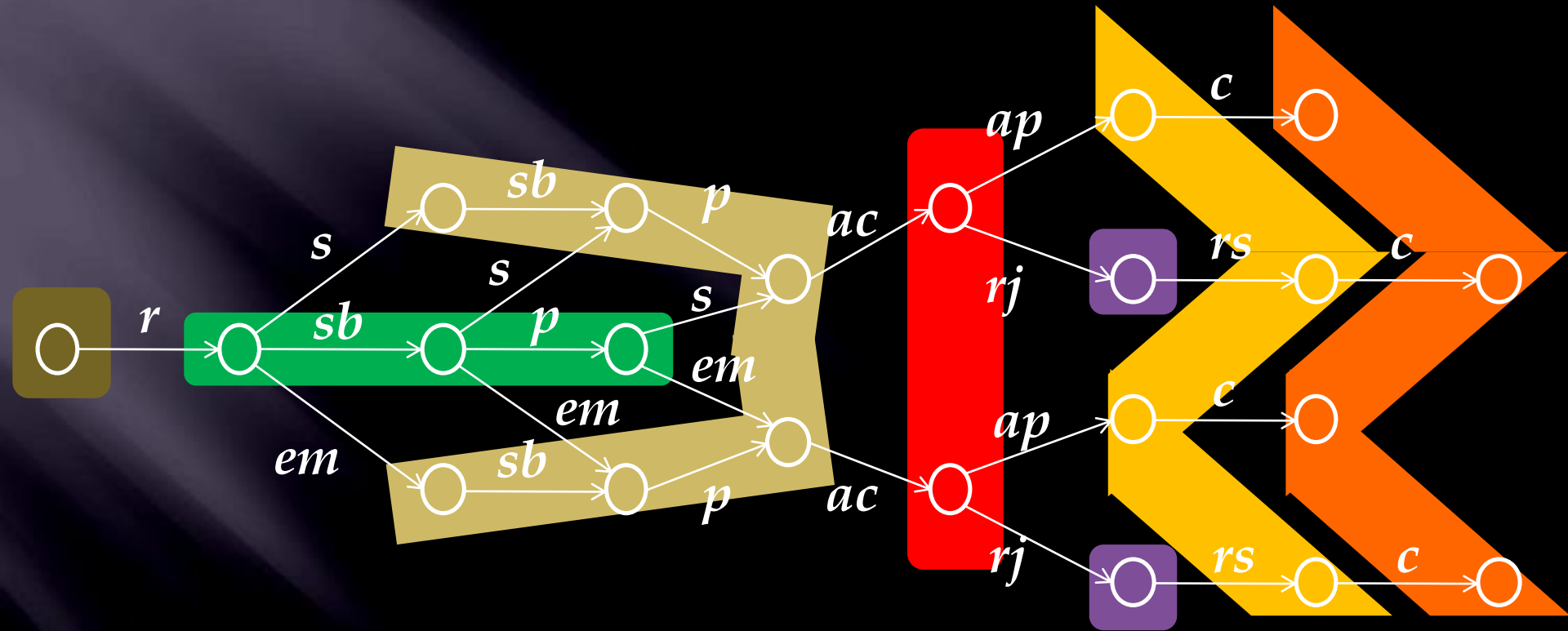
Decompositional Strategy



em, s not covered!

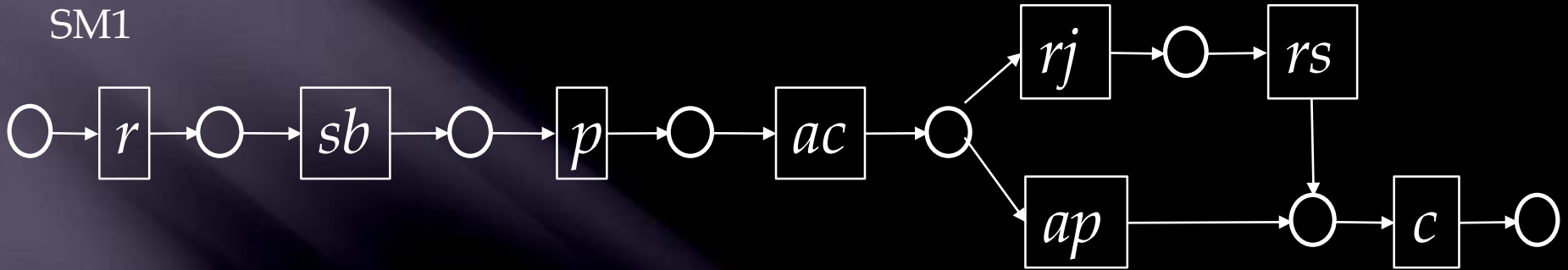


Decompositional Strategy

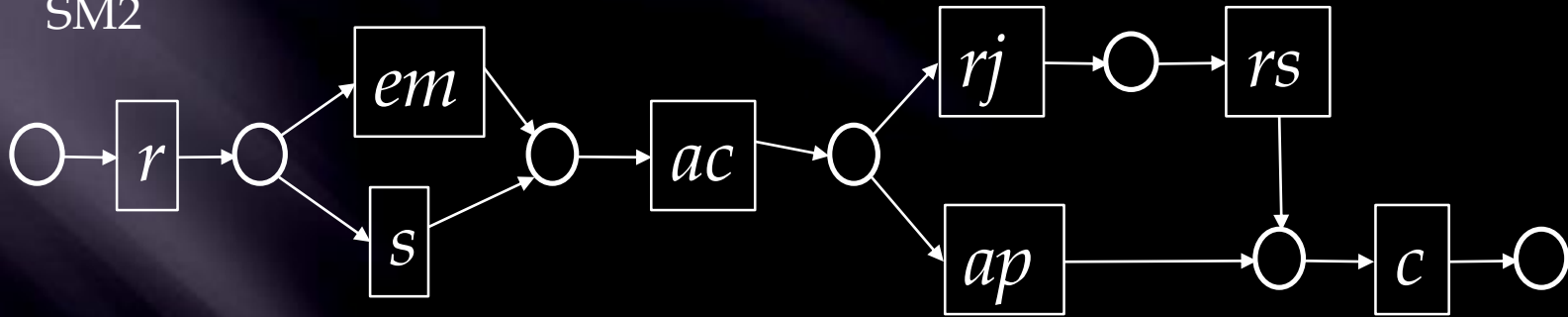


Decompositional Strategy

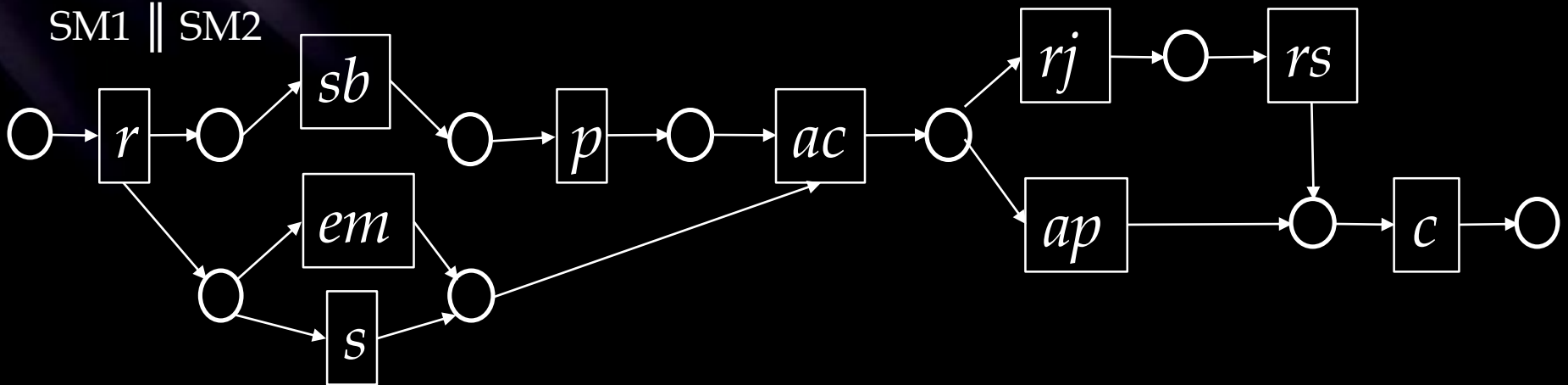
SM1



SM2



SM1 || SM2



Decompositional Strategy

General Algorithm:

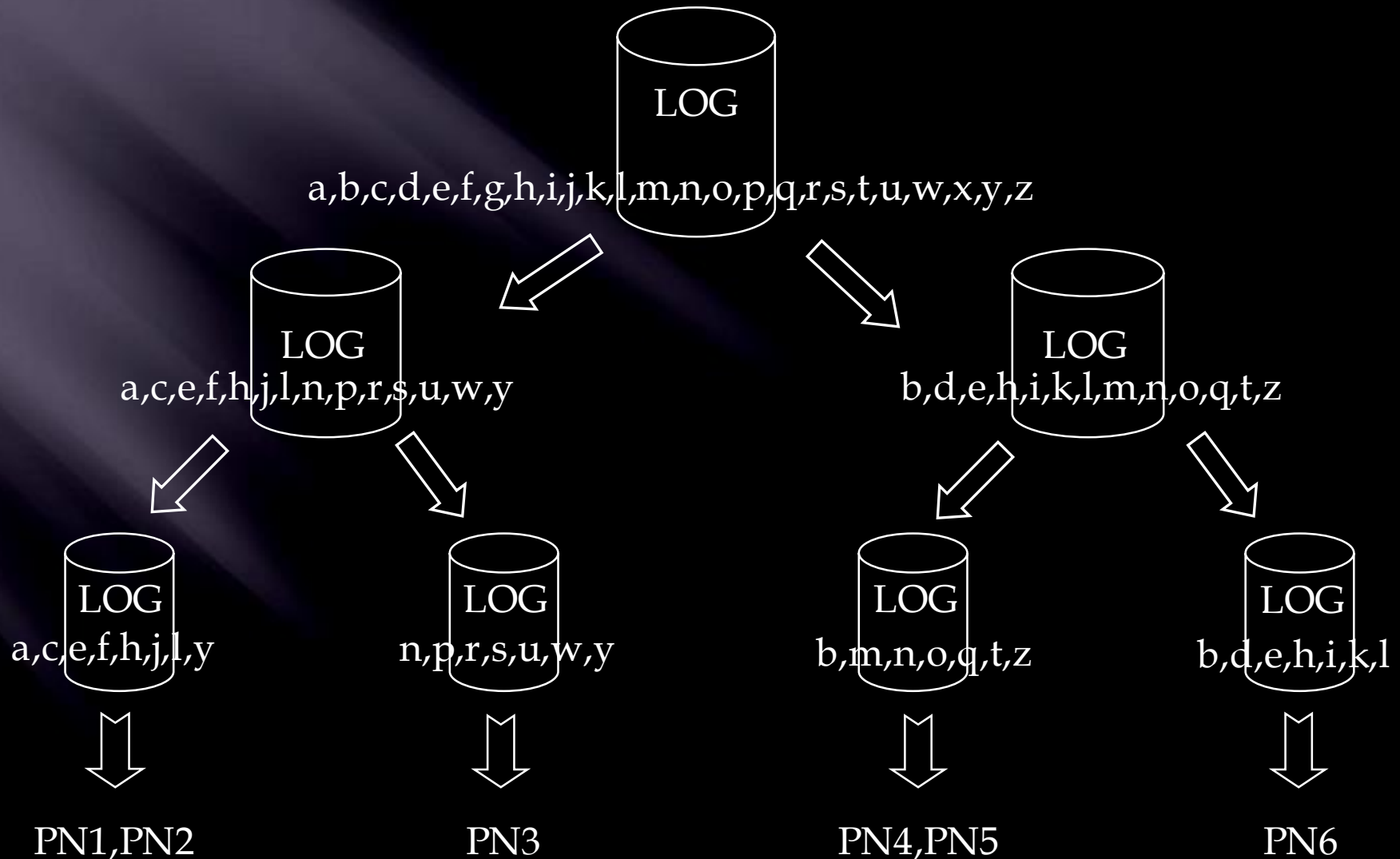
```
while (there is still event  $ev$  uncovered) do  
  find  $SM_i$  that covers  $ev$ ;  
   $i = i + 1$ ;  
  update covering table  
endwhile
```

Theorem: $L(TS) \subseteq L(SMC_1 \parallel \dots \parallel SMC_n)$

Outline

- ▣ Region-based approach for Process Mining
- ▣ Decompositional strategy: sequential views
- ▣ **Divide-and-Conquer strategy**

Divide-and-Conquer Strategy

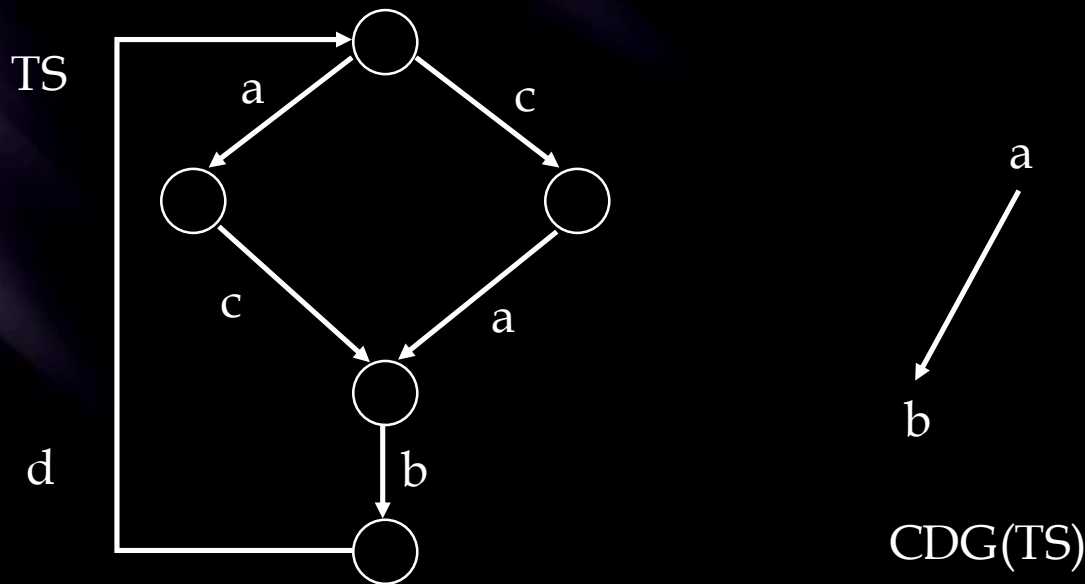


Event partition

- ▣ Events in the same class if:
 - Related by the log. Examples:
 - ▣ *Events performed by the same person*
 - ▣ *Events within a particular time period*
 - Causality relations: too strong notion to compute
 - Triggering relations: defined on the TS, easy to compute
- ▣ **How to obtain a meaningful partition ?**

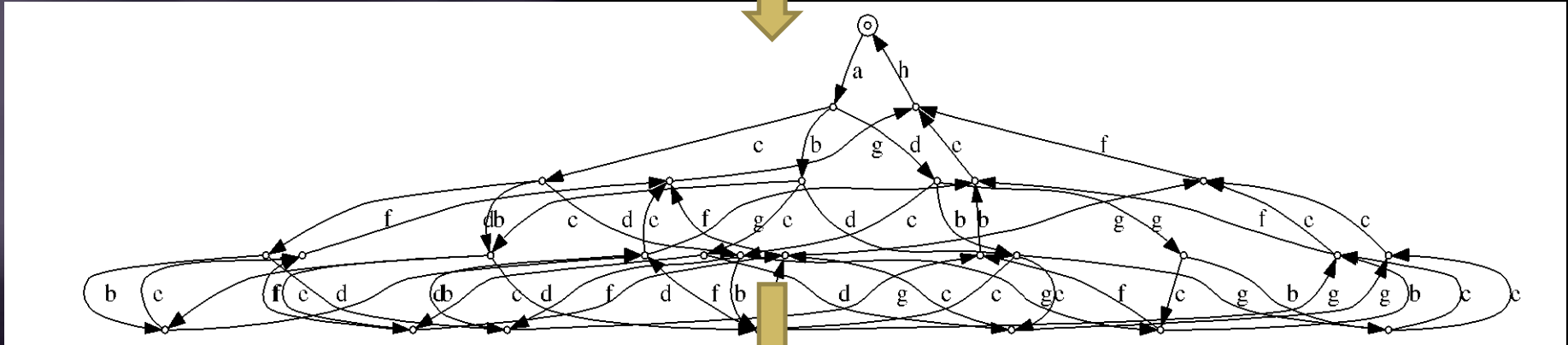
Causal Dependency Graph (CDG)

- Graph having a node per each event of the log
- An arc between two nodes x, y iff x triggers y :

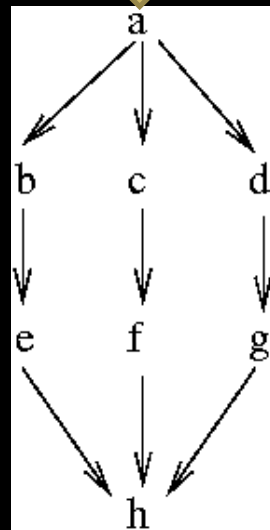


Causal Dependency Graph (CDG)

EVENT LOG



TS



CDG(TS)

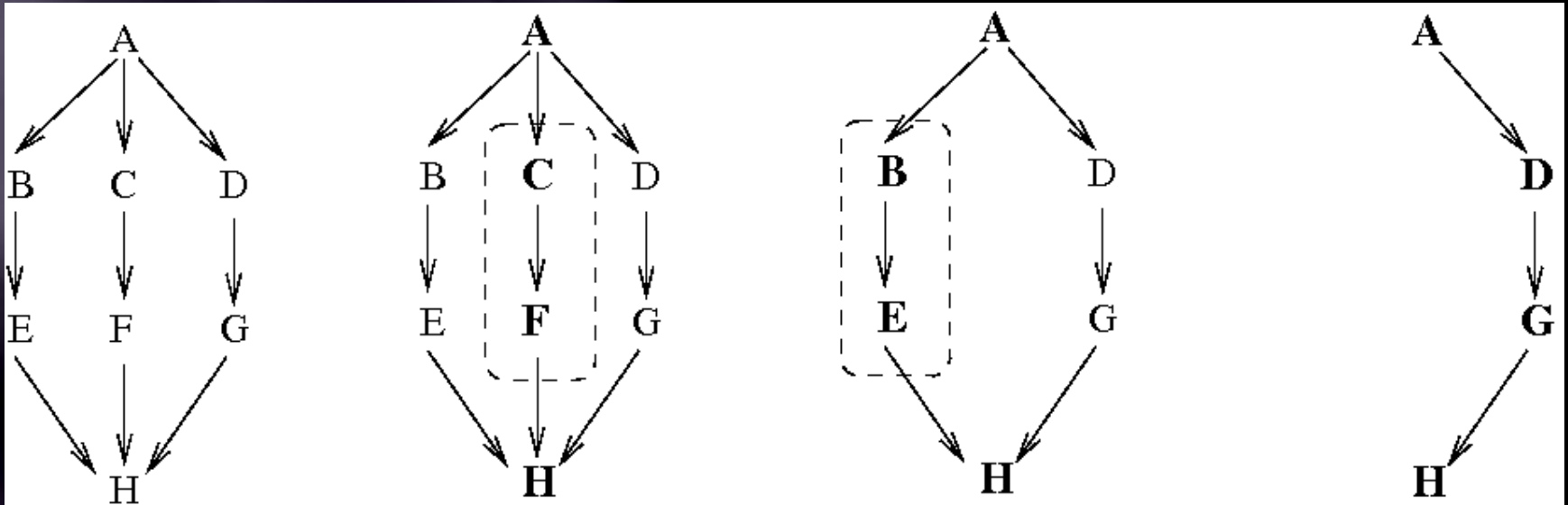
Spectral Approach

- ▣ Main idea: find *minimal cuts* in the CDG, i.e. sets of nodes tightly related between them and with few connections with the rest of the graph

- ▣ Short survey on Spectral Graph theory:

- Graph G with *adjacency matrix* A and *degree matrix* D
- $L = D - A$ is called the *Laplacian matrix* of G
- In equation $L \cdot x = \lambda \cdot x$, x is called *eigenvector*, λ is *eigenvalue*
- The *eigenvector* corresponding to the second *eigenvalue* of L is a good bi-partition (cut) of G [Cvetkovic *et al.*]
- Efficient algorithms exist to compute this vector

Spectral Approach: example

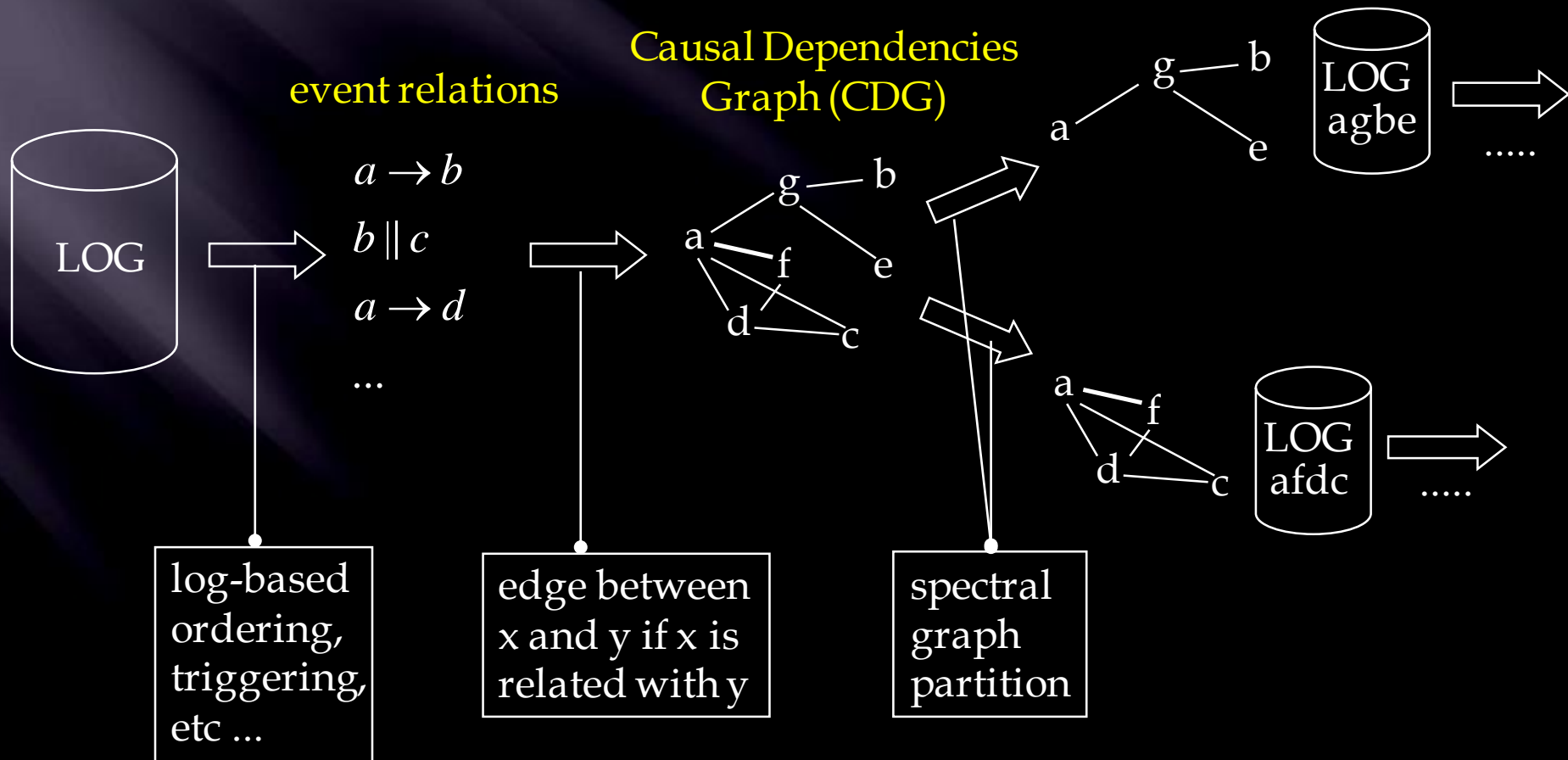


INITIAL CDG(TS)

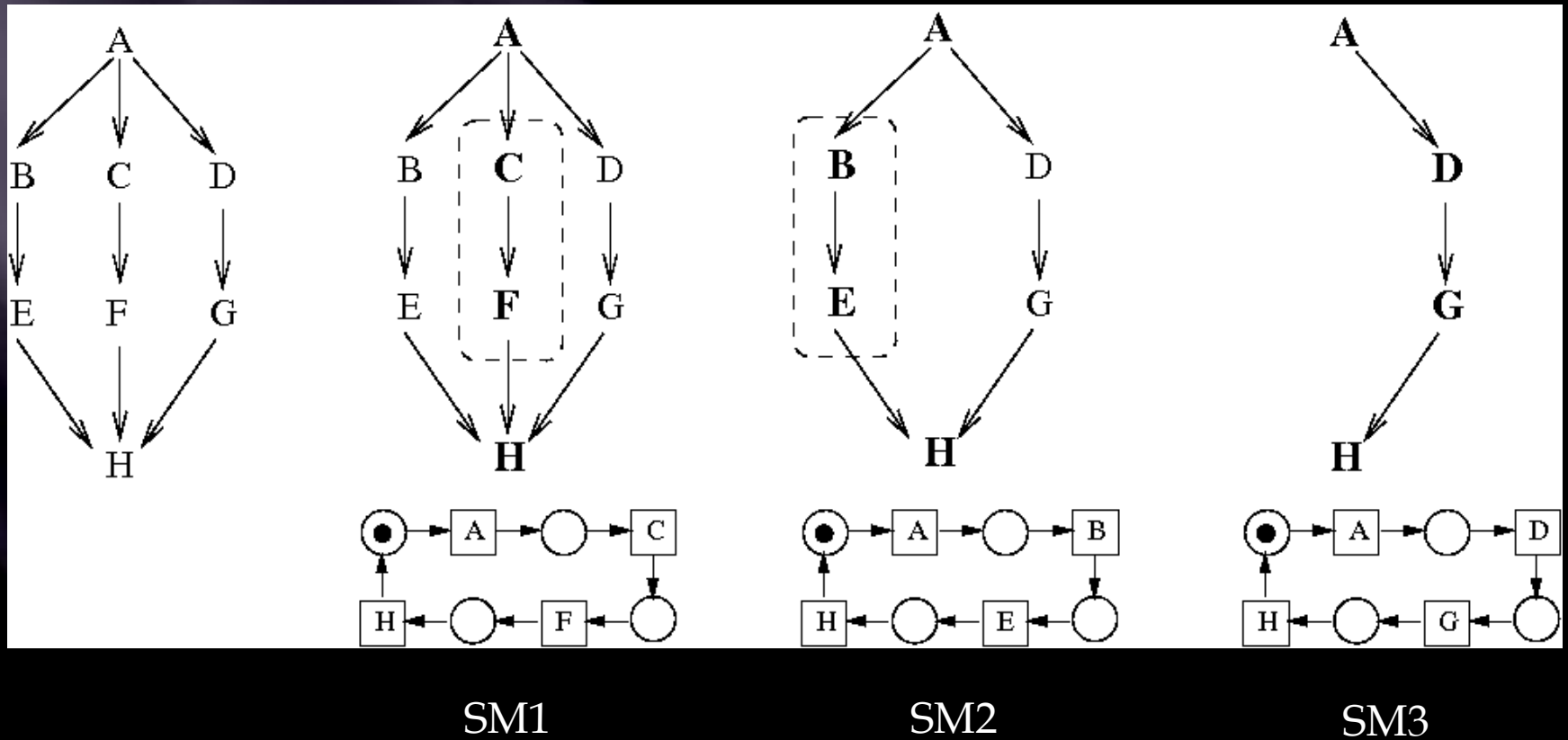
FIRST CUT

SECOND CUT

Divide-and-Conquer Strategy



Spectral Approach: example



Divide-and-Conquer Strategy

General Algorithm:

compute $CDG(TS)$;

$(E_1, \dots, E_n) = \text{FindSpectralPartition}(CDG(TS), \text{MaxSize})$;

forall E_i **do**

$E_i = E_i + \text{AddBorderEvents}(CDG(TS), E_i)$;

$\text{SMDecomposition}(TS | E_i)$

endwhile

Theorem: $L(TS) \subseteq L(SMC_1 \parallel \dots \parallel SMC_n)$

Thanks!